

## LCA Methodology

# Allocation of Environmental Burdens in Co-product Systems: Product-related Burdens (Part 1)

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**Abstract.** This paper uses an industrial case study of a boron system producing five co-products to examine different allocation methods recommended by ISO 14041 and compare them with the allocation methods most commonly used by LCA practitioners. In particular, allocation by physical causality is discussed. The paper illustrates how the use of whole system modelling can help to identify the correct type of causality for allocation. The case examined here concerns marginal changes of product-related parameters in the system, in this case represented by the output of boron co-products. The analysis shows that in some cases it can be correct to allocate the burdens on the basis of a simple physical quantity, such as mass, as long as the allocation parameter is based on physical causation and is not chosen arbitrarily. In whole system modelling, the correct causality is identified by the model itself, so that the possibility of allocation by an arbitrary parameter is avoided. However, as for system disaggregation and expansion, allocation through mathematical modelling may only be possible if detailed data for the system are available.

**Keywords:** Allocation; boron; environmental impacts; LCA; Life Cycle Assessment; linear programming; marginal values; system analysis

## 1 Introduction

Many aspects of the Inventory phase of Life Cycle Assessment (LCA) are essentially the same as the approach to material and energy balances used routinely in process engineering. However, allocation stands out as a recognised methodological problem specific to LCA. Allocation is the process of assigning to each of the functions of a multiple-function system only those environmental burdens associated with that function. The allocation issue arises because of a feature of LCA not common to conventional process system analysis: in LCA, economic activities are usually shared among a number of different supply chains, whereas in conventional process analysis operations within a process are dedicated to that process, even though it may produce a number of co-products (see CLIFT and AZAPAGIC, 1999).

This paper illustrates how modelling approaches used routinely in process system analysis can be used to address the problem of allocation in LCA. The particular modelling

technique used here is linear programming which is well adapted to use in LCA (AZAPAGIC, 1996; AZAPAGIC and CLIFT, 1994, 1995, 1998, 1999a-b). However, the conceptual approach to allocation illustrated here can be used with any other form of system model (CLIFT et al., 1998; 1999). The method is illustrated by applying it to a "cradle to gate" analysis of a mineral processing facility producing five boron co-products.

## 2 Allocation

### 2.1 General approaches

The problem of allocation arises in multiple-input (waste treatment processes), multiple-output (co-product systems), and multiple-use systems (open-loop recycling) (CONSOLI et al., 1993; AZAPAGIC and CLIFT, 1999a). The ISO 14041 standard (ISO, 1998) on Inventory Analysis proposes the following three-step hierarchy for dealing with allocation in multiple-function systems:

- i. If possible, allocation should be avoided by:
  - expanding system boundaries to include the additional functions related to the system under study, or
  - disaggregating the system and identifying the subsystems which are not common to all or some of the functional units;
- ii. If that is not possible, then the allocation problem must be solved by using physical causation which reflects the underlying physical relationships among the functional units;
- iii. Where physical relationships cannot be established, other relationships, including the economic value of the functional outputs, can be used.

ISO 14041 recommends that each study should follow this procedure, starting with step (i) and then moving to steps two and three if necessary. However, it is clear that this will not be possible in all cases. One of the reasons is that the method to be used for allocation will, in general, depend on the goal of the study. For instance, the goal could be comparison of a product with the same or an alternative product produced in a different system. In other instances, the goal could be to compare the co-products in the same system or to provide LCA data for the co-products which are

subsequently used in different product systems. The applicability of the ISO allocation procedure to these different study goals is discussed in this paper, based on the specific example of a system producing boron co-products. However, given the actual goal of the study in this work, particular attention is given to step (ii) to show when it is appropriate to use physical causality for allocation and how this can be carried out.

## 2.2 Avoiding allocation

Amongst others, Tillman et al. (1994) have discussed the use of system expansion to avoid allocation. Referring to the specific case of co-product systems, there are two essentially equivalent system expansion approaches, both of which define:

- System I, producing a functional output of interest together with other co-products in the system,
- System II, producing only the functional output of interest, and
- System III, producing all co-products from System I except the functional output of interest.

The first, direct system enlargement, adds System III to System II, so that the comparison is now between System I and Systems II+III. The second approach, known as the "avoided burdens" approach, subtracts from System I the environmental burdens from the product System III. Thus, the functional output of interest in System I is compared with the functional output from System II.

Possible applications of these approaches to the boron system are discussed in Section 3. Both system enlargement and the avoided burdens approach are useful for comparing alternative ways of delivering a functional output, particularly when the number of co-products is small. For example, the "avoided burdens" approach is useful in cases where output of co-products from one system displaces their production elsewhere, as in recovery of materials or energy from waste (LINDFORS et al., 1995; DOIG and CLIFT, 1995), or in co-generation of electricity and thermal energy. The cogeneration case is illustrated for the boron case study later in this paper.

However, the system expansion approach in general relies on the existence of alternative ways of producing the co-products and the availability of inventory data for the alternatives. The other way to avoid allocation recommended by ISO 14041 (1998) is to disaggregate a multiple-function system by examining it in sufficient detail to distinguish among different subprocesses and split off those which are relevant to one functional output only. This approach is only applicable to a system for which detailed process data are available. In such a case, disaggregation should always be used to minimise the allocation problem regardless of the goal of the study. However, disaggregation alone cannot avoid allocation completely in a complex interlinked product system, like the one considered in this paper. Nevertheless, as illustrated in the case study, disaggregation is an essential part of formulating a suitable system model, which then can be used to solve the allocation problem by physical causality.

## 2.3 Allocation by physical causality

Where it is not possible to avoid allocation, either because of the structure of the processes being considered or because of the goal of the study, the ISO procedure requires allocation to be based on "physical causality". The common practice of arbitrarily choosing some simple parameter for allocation, such as mass flow, is therefore explicitly abandoned. This approach thus adopts the recommendations made by a number of researchers (e.g. AZAPAGIC and CLIFT, 1994; 1995; CLIFT et al., 1998, 1999) that allocation should be based on causal relationships established through system modelling. It is also clear from the ISO 14041 statement that physical causality is to be interpreted broadly, to cover any physical, chemical, biological or technical relationships which relate the environmental burdens to the functional outputs from the system. These relationships provide the basis for formulating "a mathematical model of the system (as) a set of mathematical relationships which represent an abstraction of the real system under consideration" (FLOUDAS, 1995). Thus, following ISO 14041 (1998), allocation by physical causality must be a result of mathematical system modelling.

The implications of this have been explored in more detail elsewhere (e.g. CLIFT and AZAPAGIC, 1995; AZAPAGIC and CLIFT, 1999a; CLIFT et al., 1998). In general, it follows that allocation of burdens among different functions of the system depends both on the way the system is operated and on the kind of change which is the subject of the LCA study. System structure and operation dictate the form of the system model; for example whether the operation of the system can be described by linear or nonlinear relationships. The changes to be considered in a co-product system may take the following form (CLIFT and AZAPAGIC, 1995; CLIFT et al., 1998; 1999):

- *marginal*, considering infinitesimal change about the existing operation of the system; for example infinitesimal variations in a functional output;
- *incremental*, corresponding to a shift to a new operating point; for example involving a small but finite change in one or more functional outputs;
- *average (or discrete)*, representing a major shift in the operation of the system; for example eliminating one or more functional outputs completely.

In addition, the time frame over which the change is contemplated may affect the scenarios which the mathematical model aims to represent. The specific allocation case developed in this paper refers to short-term marginal changes in the operation of the system, including marginal variations in the co-product outputs.

The environmental burdens from a system depend both on product- and process-related parameters. It is therefore useful to distinguish between product- and process-related burdens (CLIFT and AZAPAGIC, 1995; DOIG and CLIFT, 1995; AZAPAGIC and CLIFT, 1999a). For instance, product-related parameters include product output or material throughput while process-related parameters may be temperature or pressure of an operation in the system. Product-related parameters can particularly be relevant for providing LCA data for users of one of the co-products, or for guiding environ-

mental management of the entire product system (AZAPAGIC, 1996; CLIFT et al., 1998), whereas process-related parameters are mainly of interest for environmental management of a process or plant. Product-related burdens are illustrated later in this paper by the boron case study. Discussion on process-related burdens can be found in Azapagic and Clift (2000).

Allocation based on physical causality and system modelling relies on being able to vary system parameters independently (AZAPAGIC and CLIFT, 1994; 1999a; CLIFT et al., 1998). For instance, in cases where the interest lies in product-related burdens, it must be possible to change the output of the co-products independently of each other. Where the ratio of outputs is fixed, for example by stoichiometry as in production of chlorine and caustic soda, then allocation based on physical causality is impossible. For these cases, it is necessary to resort to option (iii) in ISO 14041 (1998) and use other relationships, such as economic value of the outputs, as the basis for allocation. However, it is important to note that this approach cannot be used arbitrarily but only where demanded by the characteristics of the product system. The analysis of the boron system given below provides an example of how misleading the results of allocation can be if mass flow or economic value are chosen arbitrarily as allocation parameters, without considering causal relationships in the system.

### 3 Allocation in the Boron Co-Product System

The multiple-function system used here as an illustration is selected because it includes examples of the principle problems in allocation discussed above. It is an existing mining and mineral processing facility which produces five boron co-products:

1. disodium tetraborate decahydrate ( $\text{Na}_2\text{B}_4\text{O}_7 \cdot 10\text{H}_2\text{O}$ ) or "10 Mol" (10 Mol),
2. disodium tetraborate pentahydrate ( $\text{Na}_2\text{B}_4\text{O}_7 \cdot 4.67\text{H}_2\text{O}$ ) or "5 Mol" (10 Mol),
3. orthoboric acid ( $\text{H}_3\text{BO}_3$ ) or boric acid (BA),
4. disodium tetraborate ( $\text{Na}_2\text{B}_4\text{O}_7$ ) or anhydrous borax (AB), and
5. boric oxide ( $\text{B}_2\text{O}_3$ ) or anhydrous boric acid (ABA).

The LCA flow diagram of the boron system is shown in Fig. 1. The system is divided into foreground and background subsystems. The foreground is defined as the set of processes directly affected by the study (DOIG and CLIFT, 1995; CLIFT et al., 1998), delivering a functional unit specified in Goal and Scope Definition. The background is that which supplies energy and materials to the foreground system, usually via a homogeneous market so that individual plants and operations cannot be identified. Further discussion is related to allocation in the foreground only; the data for the background originate from a database and the allocation approaches (usually by mass basis) used there have been adopted for this case study.

The foreground system comprises the boron mine and plant, while the background encompasses all other activities from extraction of raw materials to their delivery to the foreground system. In the foreground, two boron minerals, borax and

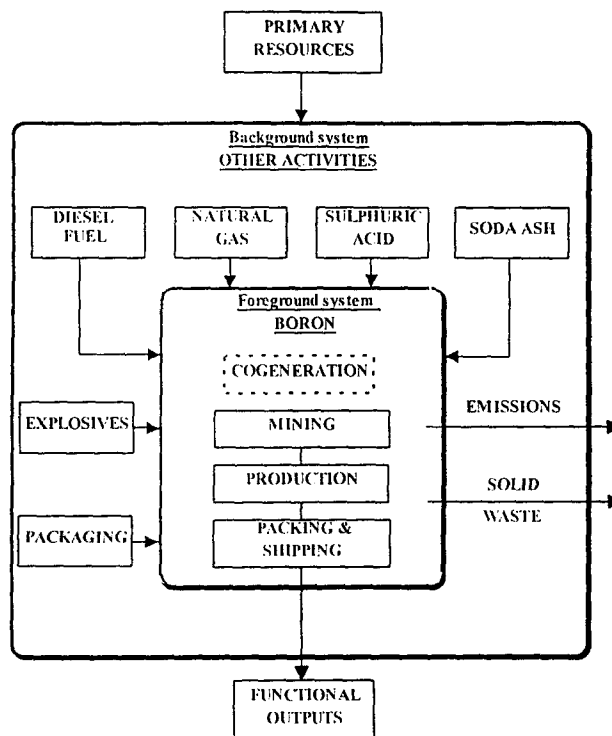


Fig. 1: LCA flow diagram of the boron system

kernite are extracted in the mine, crushed and transported to the adjacent plant. 5 and 10 Mol borates are produced by dissolving borax and kernite in water; BA is produced in a separate plant, by reacting kernite ore with sulphuric acid. AB and ABA are made in high-temperature furnaces from 5 Mol borate and BA, respectively. All products are then either packed or shipped in bulk. Electrical energy and the steam for the system are provided by the on-site natural gas Cogeneration plant, which meets all of the electricity and most of the steam demand with additional steam provided by the steam plant, also fired by natural gas. All activities, from extraction of raw materials to production and packing of the boron products and materials used, are included in the system described here. However, the use and disposal phases of the products are not considered, making this essentially a "cradle to gate" study. A more detailed account of this system is given in Azapagic (1996) and Azapagic and Clift (1999b).

This study had two goals:

1. to evaluate the environmental performance of the system from "cradle to gate" as a guide for environmental management, and
2. to provide background LCA data for other systems using the boron co-products.

Depending on the goal, three types of functional unit can be defined ( $\rightarrow$  Table 1). The first, defined as "operation of the system for one year" and represented by the annual output of the five boron products, is relevant for the first goal of the study. The other two types of the functional unit are appropriate for the second goal and can be defined either as 1000 kg of each product or 1000 kg of  $\text{B}_2\text{O}_3$  equivalent in

**Table 1:** Functional units in relation to the goal of the study

	Study goal: 1	Study goal: 2	Study goal: 2
Functional unit $\Rightarrow$	"Operation of the system for one year" (t/yr)	"1000 kg of each product" (kg)	"1000 kg of B <sub>2</sub> O <sub>3</sub> equivalent" (kg)
Co-products $\Downarrow$			
10 Mol	81200	1000	2740
5 Mol	813640	1000	2090
BA	148100	1000	1780
AB	16420	1000	1445
ABA	5260	1000	1000
<b>Total</b>	<b>1064620</b>	-	-

each product. The former is applicable to a comparison of the same products delivering the same function but produced in different product systems. The functional unit based on B<sub>2</sub>O<sub>3</sub> equivalent can be used when comparing two different products with the same function, produced either in the same or different systems. The relevance and application of the three ISO 14041 allocation methods to these goals is discussed in the following sections.

### 3.1 Avoiding allocation

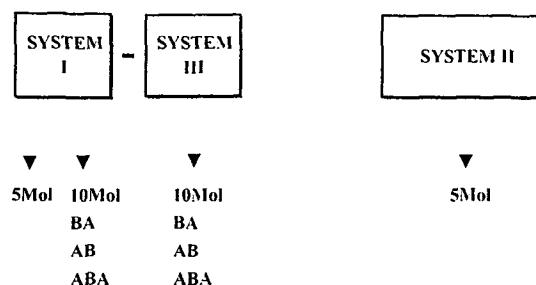
If the goal of the study is comparison of one of the co-products from the boron system (System I in Fig. 2), for example 5 Mol borate, with 5 Mol (or an alternative product) produced in another, single-output system (System II), then allocation can be avoided in the two ways outlined in Section 2.2. System II can be enlarged so that an alternative way of producing the other co-products from System I (10 Mol borate, BA, AB and ABA) is added to System II. The comparison is now between System I and System II + III.

While this approach may be feasible for simpler systems producing at most two co-products, it becomes impractical for more complicated systems such as the one considered here. It would involve gathering a significant amount of additional data on alternative production of the co-products which are not readily available and in some cases are non-existent. Therefore, in cases like this, direct system enlargement is not applicable.

Furthermore, this method does not provide data on the contribution of individual co-products to the total environmental burdens and impacts; it only enables comparison of one system with other product systems. Therefore, avoiding allo-

cation by expanding system boundaries is not relevant for the second goal study, i.e. to provide background LCA data for other systems using one or more of the boron co-products.

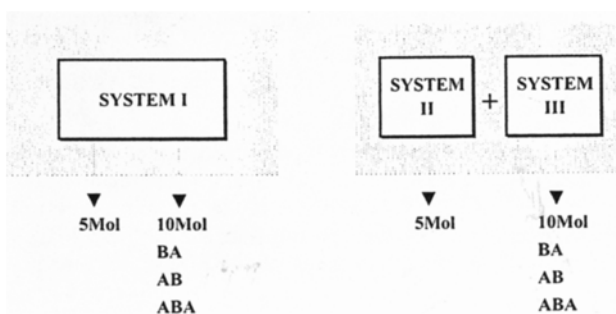
As discussed in Section 2.2, an alternative but equivalent approach is the "avoided burdens" method. Here, the burdens arising from an alternative way of producing the co-products in System III (i.e. 10 Mol, BA, AB and ABA) are subtracted from the burdens in System I. In this case, 5 Mol in System I is compared to 5 Mol in System II, as illustrated in Fig. 3. However, for such a complex system, the "avoided burdens" approach is impractical for the same reasons as the approach in Fig. 2. This method is further discussed later in this paper using the example of the Cogeneration plant.

**Fig. 3:** Avoiding allocation by the "avoided burdens" approach

Another way to avoid allocation in the boron system, as recommended by ISO 14041 (1998), is to disaggregate it and split-off the processes that are dedicated to one functional output only. For instance, the boron system can be disaggregated into subprocesses that are dedicated for production of BA, AB and ABA only. However, production of 5 Mol and 10 Mol cannot be disaggregated because their production lines are coupled from early in the process. This is an example of a case where disaggregation cannot be used to avoid allocation.

### 3.2 Allocation by physical causality: Product-related burdens

This paper is concerned primarily with the co-product system in which physical causality can be used as a basis for allocation and where the marginal changes in the system are of interest. This approach is known as marginal allocation

**Fig. 2:** Avoiding allocation by system enlargement

(AZAPAGIC, 1996; AZAPAGIC and CLIFT, 1994; 1998, 1999a). To model the physical relationships in the boron system and determine the marginal allocation coefficients on the basis of physical causality, Linear Programming (LP) has been used as a specific modelling tool. However, system modelling in this context is not limited to LP; other modelling techniques can also be used for the same purposes. For the ease of following the discussion in the subsequent sections, the next paragraph introduces some of the vocabulary of LP. The LP model and definition of the marginal allocation coefficients are given in the Appendix (p. 367).

In LP, the relationships that describe the system behaviour are called "constraints". At the solution of the LP model, total burdens are calculated together with the so-called "marginal values", which show the contribution of each constraint to the total burdens. Thus, the marginal values represent the burdens "allocated" to the constraints. At the solution, the constraints can be "active" or "non-active", as determined by the operating state of the system. If they are active, then they limit the operation of the system and their marginal burdens will be non-zero, whereas the non-active constraints have zero marginal burdens. Since the constraints are related to either product or process parameters, it follows that the burdens allocated to the active constraints are either product- or process-related. For instance, if the operation of the system is limited by the capacity of a process, the burden will be allocated to this process and will be process-related. On the other hand, if the output of the co-products solely determines the operation of the system, then the burden will be allocated to the products and will thus be product-related. This paper considers the case where the goal of the study is to analyse the effect on burdens of marginal changes in the product-related parameters, here defined by the product outputs. Although we refer here to the burdens only, the same kind of analysis applies to the impacts (see the Appendix). Thus the focus here is on product-related burdens. Process-related bur-

dens and their change with the operating state of the system are analysed in Azapagic and Clift (2000).

The burdens, defined by eqns. (10) in the Appendix, are calculated together with the marginal allocation coefficients at the solution of the boron LP model. As the product-related burdens are of concern, the only active constraints at the solution are those which describe the co-product output [eqns. (6)]; the process-related constraints [eqns. (7)-(9)] are non-active and their marginal allocation coefficients are therefore zero. The marginal coefficients of the active product constraints (6) relate the burdens to the product outputs [through eqns. (17)]; the resulting values are shown in Fig. 4. To preserve confidentiality, the marginal allocation coefficients are shown in relative terms. The figures show that the highest burdens are allocated to the ABA and AB, mainly because of the energy intensive processes by which they are produced. 10 and 5 Mol borates are the most preferred products in the boron system in environmental terms.

For further discussion of the marginal allocation approach, two specific burdens will now be considered. The results of the marginal approach are also compared with the two other most commonly used bases for allocation: mass and financial value. Although the discussion is based on the functional unit defined as the "operation of the system for one year", the marginal coefficients, and therefore relative contributions of the products to the burdens, remain the same for the other two functional unit definitions given in Table 1.

### 3.2.1 Carbon dioxide

Fig. 5 shows the results of marginal allocation of the CO<sub>2</sub> emissions among different boron products. ABA and AB account for most of the total CO<sub>2</sub> emissions of 2.52920 t/yr, with the marginal allocation values equal to 1.942 and 0.980 t/t, respectively. This means that, for instance, producing one additional tonne of ABA will cause a total increase of

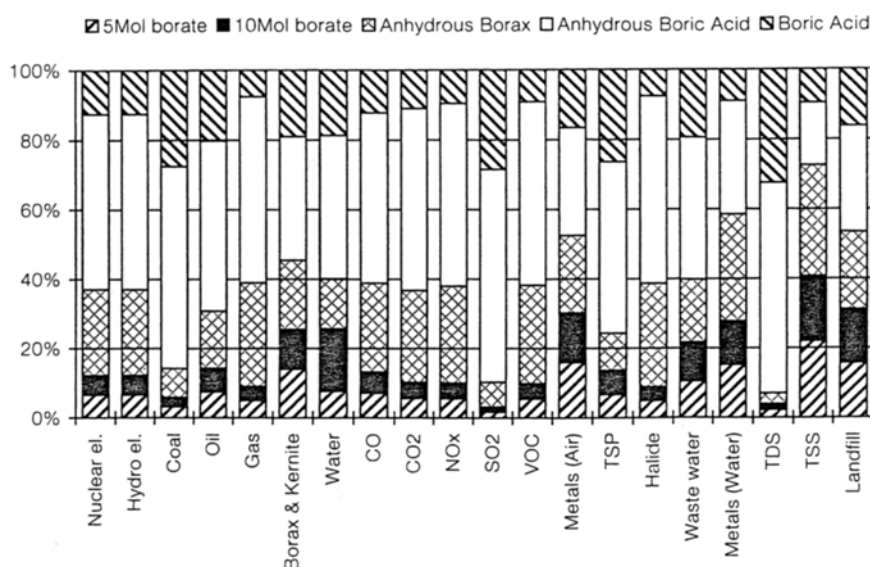


Fig. 4: Environmental burdens allocated by the marginal approach

CO<sub>2</sub> emissions of 1.942 tonnes. These high values arise from the energy intensive processes involved, and the life cycles of BA and 5 Mol which are used as raw materials in these processes. The CO<sub>2</sub> emissions allocated to other products are considerably smaller, and range from 0.414 t/t for BA to 0.206 t/t and 0.176 t/t for 5 Mol and 10 Mol, respectively. For this case, where the interest lies in marginal changes in the burdens with product outputs, the burdens are fully allocated to the products. This is in agreement with the recommendation of ISO 14041 (1998) that the sum of the allocated inputs and outputs should be equal to the total unallocated inputs and outputs.

Fig. 5 also compares marginal allocation with results of using other bases for allocation: mass, B<sub>2</sub>O<sub>3</sub> content and market value. To illustrate the importance of system disaggregation, allocation on the mass basis for both aggregated and disaggregated systems is also compared in Fig. 5. If allocation by mass is done without system disaggregation, the total burdens are allocated among different products in proportion to the mass of their outputs; all products in the boron system then have the same allocation factor of 0.238 t/t (total CO<sub>2</sub> emissions of 252920 t/yr divided by the total yearly production of 1064620 t/yr). The difference between this and the marginal allocation coefficients ranges from 13% for 5 Mol to 85% for ABA. The reason for this is that allocation on the mass basis without system disaggregation fails to account for differences among the processes for production of the co-products. This example illustrates how misleading the allocation results can be if they do not reflect the underlying physical causality.

A similar conclusion emerges if allocation is based on the B<sub>2</sub>O<sub>3</sub> content<sup>1</sup> in different products. Although it seems that

the B<sub>2</sub>O<sub>3</sub> allocation coefficients increase as the degree of processing of different products increases to give a higher boron content (→ Fig. 5), this increase is only linearly related to the content of B<sub>2</sub>O<sub>3</sub> in the products and not to the burdens associated with their processing. Furthermore, allocation on the basis of market value<sup>2</sup> does not give the correct results either. Although the burdens allocated on the marginal and market value bases are quite similar for 10 Mol, 5 Mol and BA, the difference is much larger for AB and ABA (51% and 38%, respectively). This implies that the "external" costs of the environmental burdens are not proportional to the current economic values of these products. Thus, allocation by financial value can give misleading results and should not be used in systems where physical causality exists.

However, the results are quite different if allocation by mass basis is done after the system has been disaggregated to take into account differences in the processes for producing different boron products. Here, the disaggregation is carried out by splitting-off production of BA, AB and ABA, as separate processes, from production of 5 and 10 Mol. Although the production lines of 5 and 10 Mol borates are coupled at the beginning, they separate later in the process after which the system can be disaggregated further. However, for the parts of the process common to the two products, further disaggregation is not possible and, for purposes of the illustration here, the burdens have been allocated by mass. Fig. 5 shows that marginal allocation and allocation by mass with system disaggregation give the same results. This means that in case of the CO<sub>2</sub> emissions, physical causation can be represented by a simple physical quantity, i.e. mass. Therefore, in some cases it may be correct to allocate the burdens on the basis of a physical quantity; however, the point here is that the choice

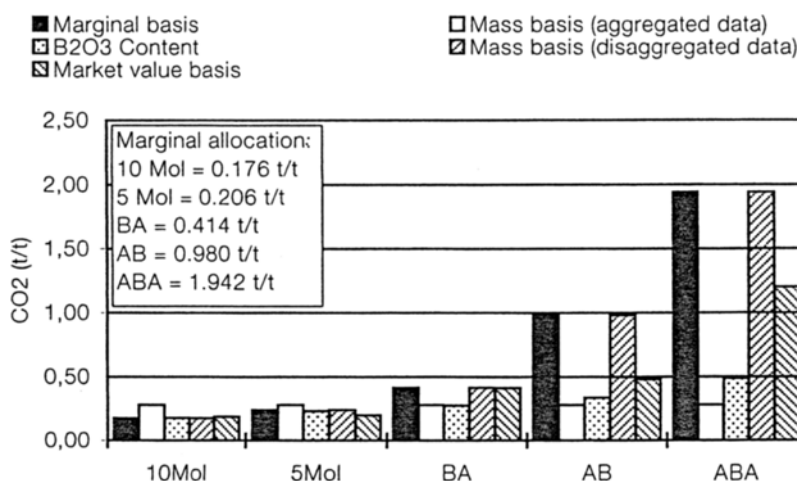


Fig. 5: Comparison of different allocation methods for CO<sub>2</sub> emissions

<sup>1</sup> The allocation coefficients are obtained by dividing the total CO<sub>2</sub> emissions of 252920 t/yr by the total output of B<sub>2</sub>O<sub>3</sub> equiv. of 518605 t/yr and multiplying the result by the stoichiometric ratio of the molecular weights of B<sub>2</sub>O<sub>3</sub> and the product for which the allocation coefficient is calculated. For instance, for 10 Mol, the allocation coefficient based on the B<sub>2</sub>O<sub>3</sub> content is equal to:  $(m_{CO_2}/m_{B_2O_3 \text{ equiv.}}) \cdot (M_{B_2O_3}/M_{10Mol}) = (252920/518605) \cdot (139.2/381.2) = 0.178$ .

<sup>2</sup> The market value is here taken to be the gross selling price in 1994 per tonne of product: 10 Mol = \$234; 5 Mol = \$255; BA = \$527; AB = \$613; ABA = \$1535.

of allocation parameter is based on the physical causation involved and is not arbitrary. A correct type of causality can only be identified if the system operation is well understood and detailed data on subprocesses in the system available. In whole system modelling, for example using LP, the type of causality is identified by the model itself, so that the possibility of arbitrary allocation is eliminated.

### 3.2.2 Solid waste

This environmental burden is mostly related to the overburden from the mine. The burdens allocated by the marginal approach are shown in Fig. 6. ABA and AB contribute 18.71 and 13.75 t/t, respectively, to the total solid waste. The marginal allocated burdens for other products range from 9.55 t/t for 10 Mol, to 10.04 t/t for BA.

It is now interesting to compare the marginal allocation approach for this burden with the mass and market value approaches. Allocation of solid waste on the mass basis without disaggregation for 5 Mol, 10 Mol and BA gives similar results to those obtained by marginal allocation. However, the difference in the results between these two approaches for AB and ABA is respectively 30% and 50%. The reason for this is that allocation on the marginal basis for AB and ABA also includes solid waste arising from the life cycles of their respective feeds, i.e. 5 Mol and BA. In allocation on the mass basis without system disaggregation it is not possible to account for this effect. Therefore, this is another illustration of how the arbitrary approach breaks down in complex industrial systems where some of the products are used for production of other co-products in the system.

If the burden is allocated on the basis of  $B_2O_3$  content, however, the results seem to be more closely related to those of the marginal approach ( $\rightarrow$  Fig. 6). Although the  $B_2O_3$  allocation coefficients are not exactly equal to the marginal values, they at least follow the same trend. This is to be expected because the amount of the overburden is dependent on the total  $B_2O_3$  required in the process, which should be reflected in the allocation coefficients. However, the main reason for the difference in the results, particularly for 10 Mol and BA, is related to the additional solid waste associated with their respective processes. Like allocation by mass, this approach cannot reflect the differences among the processes for production of the co-products.

Similarly, allocation based on market value does not give correct allocation coefficients. The difference between this and the marginal value approach ranges from 10% for 5 Mol up to 65% for ABA. However, if the burden is allocated on the mass basis with disaggregated data, the results

are the same as in allocation on the marginal basis. This illustrates yet again the importance of system disaggregation for identification of correct causality in the system.

As illustrated by these two examples, system disaggregation with allocation by mass provides a correct approach for allocation in this case. Further analysis of the boron system shows that, where the interest lies in the product-related burdens, the same causality exists for the other burdens and impacts (AZAPAGIC, 1996). These findings now beg a question: why use elaborate system modelling if system disaggregation and allocation on the mass basis give the same results as marginal allocation? The answer to this is quite simple: allocation on the mass basis may hold for the state of the system where the burdens are product-related; however, if the operating conditions of the system change and the burdens become process-related, the type of causality may change and the mass basis will then be inappropriate. Since marginal allocation coefficients emerge as a result of system modelling, they embody physical or process relationships and so help identify the true causality in the system. Therefore, allocation by whole system modelling provides a more realistic approach to system analysis by being able to account for changes in the operating conditions of a system and the corresponding changes in the environmental burdens. This is discussed in Azapagic and Clift (2000).

### 3.3 Allocation in the Cogeneration plant

Although the Cogeneration plant constitutes a part of the boron products system, the lack of detailed data made it impossible to allocate the burdens in this subsystem by the marginal approach. Therefore, allocation in the Cogeneration plant is considered here as a separate example of a co-product system.

The allocation problem in the Cogeneration plant arises because electricity is exported from the system, while the cogenerated steam is used within the system boundaries. Hence, the burdens must be allocated between 443480 MW of electricity that leaves the system and 952127 t/yr of steam used within the system boundaries. System disaggregation is not possible in the case of cogeneration, because most proc-

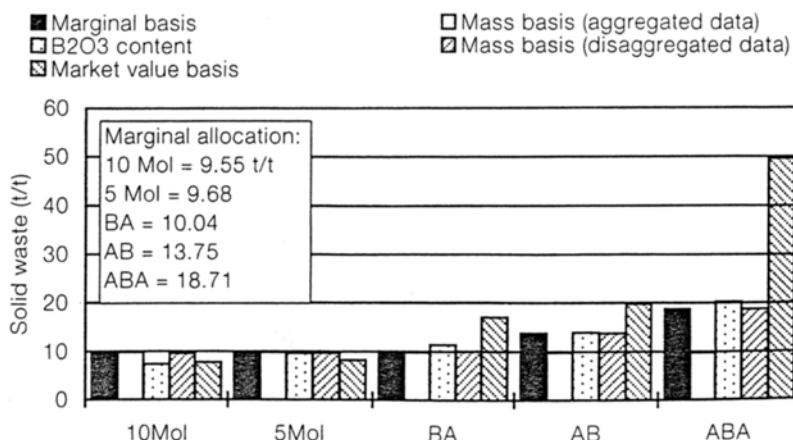


Fig. 6: Comparison of different allocation methods for solid waste

esses are common for both co-products. Allocation by physical causation is not possible either: because of the lack of detailed data it is not possible to model the system to represent the physical causalities. However, it is known that the system could in principle be described by physical causation, because the output of steam and electricity can be changed independently. Hence, following the recommendations of the ISO procedure, allocation on the basis of economic relationships should not be used. This leaves only one method, i.e. avoiding allocation by expanding system boundaries. This approach is compared here with allocation by heat content, one of the methods most commonly used in cogeneration systems.

Following on from the discussion in Section 2.2, allocation in the cogeneration subsystem can be avoided in two ways, depending on the goal of the study. If the goal is a comparison of steam production from the Cogeneration plant (System I) with steam production in another single-output system (System II), then System II can be enlarged so that an alternative way of producing electricity (System III) is added to it. The comparison is now between System I, here represented by the Cogeneration plant, and Systems II + III.

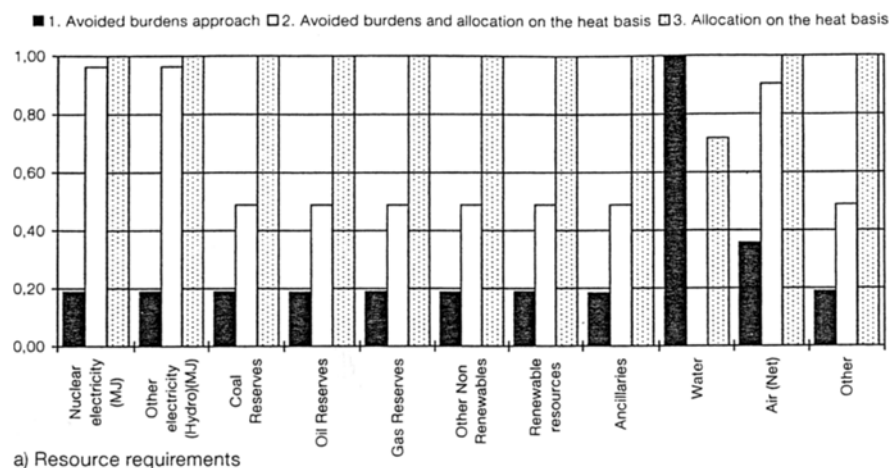
However, as noted earlier, allocation by system enlargement enables only comparison between different systems. Since in this case the interest is in calculating the burdens allocated to the steam, this approach is not relevant here. A more appropriate way of avoiding allocation by expanding system boundaries in this case is the "avoided burdens" approach. The burdens arising from the alternative production of electricity in System III are now subtracted from those

produced in the Cogeneration plant (System I) so that steam is the only output from the plant. However, before this method can be applied, it is necessary to define System III, i.e. an alternative source of electricity for this case study. Given the location of the processing facility, if electricity were not produced in the Cogeneration plant, it would be generated in an on-site power plant firing natural gas. Steam would in that case be produced in the Steam plant, also present in the foreground system. Thus, conventional power plant fired by natural gas is defined as System III.

Another possibility to allocate the burdens between steam and electricity would be to consider the burdens that are avoided by producing steam in the Cogeneration plant, thus eliminating the need for a dedicated steam generation facility, i.e. Steam plant. However, prior to applying this approach, some kind of allocation between electricity and steam produced in the Cogeneration plant is necessary. In this example, the burdens are allocated on the arbitrary basis, here taken to be heat content. This is one of the methods most commonly used for allocation in cogeneration systems, whereby the burdens are allocated by heat content of steam and electricity, with the efficiency of conversion of thermal energy taken into account (BOUSTEAD, 1992). Since energy conversion in the production of electricity is only 63% as efficient as in steam generation, this percentage of the environmental burdens is allocated to the electricity and the rest to the steam. The avoided burdens are then calculated by subtracting the burdens arising from the Steam plant from the burdens allocated to the cogenerated steam. In both systems natural gas is used as a fuel.

**Table 2:** Allocation methods for the steam cogeneration

Allocation method	Definition
<b>Method 1:</b> Avoided burdens approach	Burdens of 443480MW electricity + 952127 t steam - Burdens of 443480MW electricity by natural gas
<b>Method 2:</b> Avoided burdens approach with allocation on heat content basis	Burdens of 952127 t steam by cogeneration - Burdens of 952127 t steam by the Steam plant
<b>Method 3:</b> Allocation on heat content basis alone	$\frac{1}{3} \times (\text{Burdens of 443480MW electricity} + \text{Burdens of 952127 t steam by cogeneration})$



**Fig. 7:** Comparison of allocation methods in the steam cogeneration system



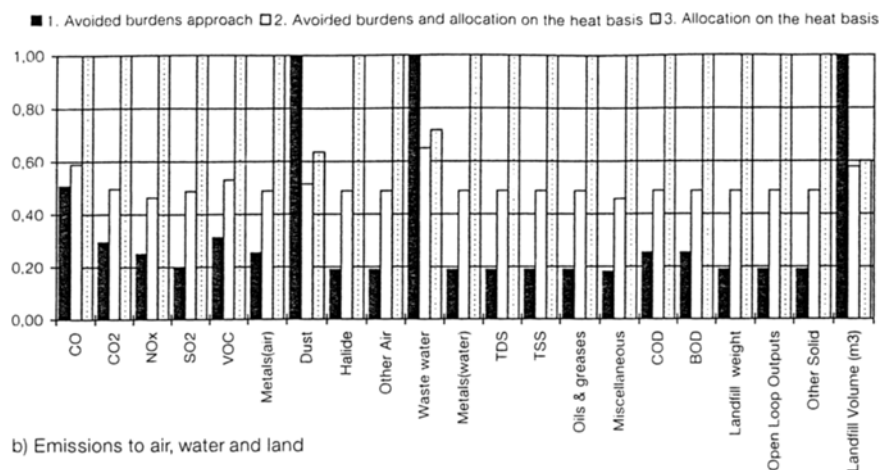


Fig. 7: Comparison of allocation methods in the steam cogeneration system (*continued*)

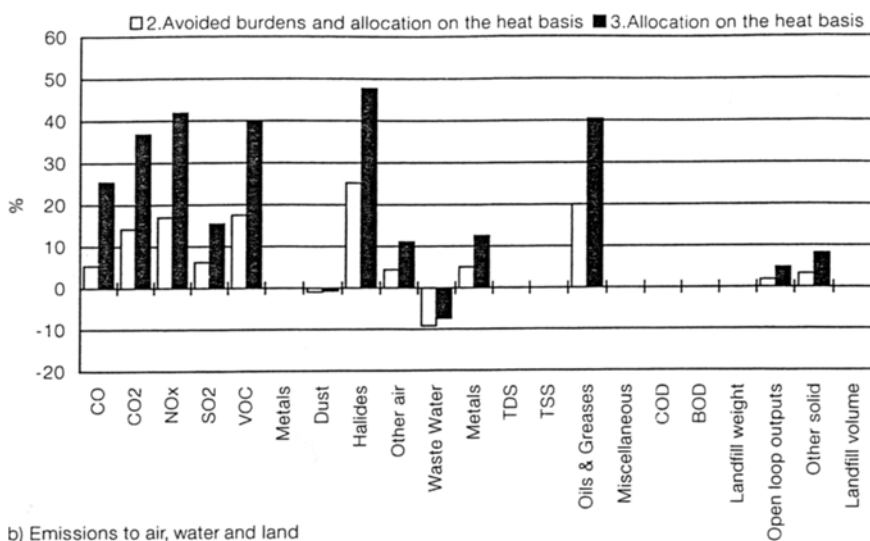
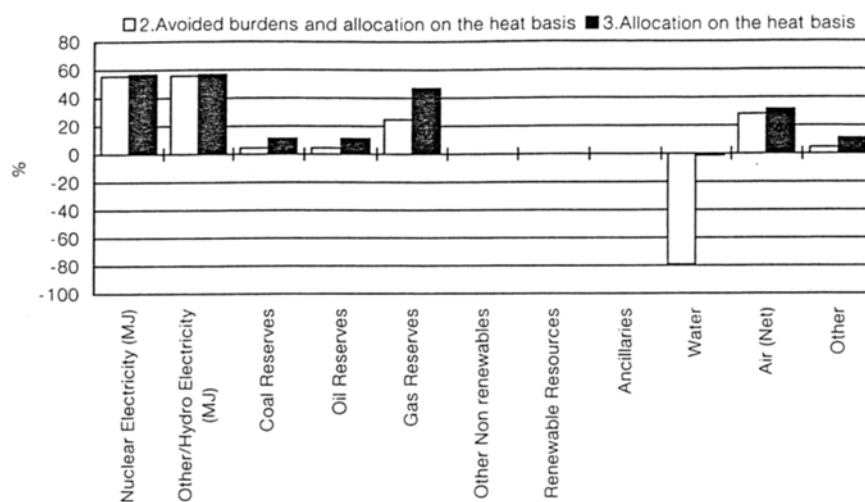


Fig. 8: Sensitivity analysis: Change of the total burdens compared to the "avoided allocation" approach

These two options for allocation by the avoided burdens approach are summarised in Table 2 (Methods 1 and 2), (p. 364) along with allocation by heat content (Method 3), and the results are compared in Figs. 7a-b (p. 364, 365). Method 1 gives on average the lowest environmental burdens because the system is credited for cogenerating heat with electricity. Method 2 also credits the system for avoiding the burdens but, like Method 3, involves allocation on the arbitrary basis of heat content.

Since the contribution of the Cogeneration plant to the total emissions from the boron system is considerable for some burdens (see AZAPAGIC and CLIFT, 1999b), it is necessary to perform sensitivity analysis to find out how different allocation methods influence the total environmental burdens. As shown in Fig. 8, the results are sensitive to the allocation method used for the cogeneration subsystem. In the resource consumption category, allocation by Methods 2 and 3 increases the use of nuclear and hydro-electricity by 60% relative to the avoided burdens approach while gas reserves allocated by these two methods increase by 30% and 50%, respectively. For emissions to air, Methods 2 and 3 increase the total burdens above Method 1 by on average 15% and 35%, respectively. The differences for the emissions to water and land are less pronounced, because the burdens from the cogeneration do not contribute much to these categories.

#### 4 Conclusions

The co-product system analysed in this paper demonstrates the importance of allocation for the outcome of an LCA study. Different allocation methods can give quite different results, and this may influence the conclusions of the study. It is thus important that, before an allocation method is chosen, different possibilities for allocation are examined and choice of the most appropriate method made so as to reflect the goal of the study. If possible, allocation should always be avoided through system disaggregation. If that is not possible, then it should be based on physical causality. Whenever feasible, causality should be described by a system model. The advantage of whole system modelling is that it examines the underlying physical causation in the system and allocates the burdens among different product- or process-related parameters accordingly. For the particular case study considered here, where the product-related burdens alone were of interest, marginal allocation gives the same results as allocation by mass with system disaggregation. Therefore, in some cases it may be correct to allocate the burdens on the basis of a simple physical quantity; however, the choice of allocation parameter must be based on the physical causation involved and not chosen arbitrarily.

#### 5 References

- AZAPAGIC, A. (1996): Environmental System Analysis: The Application of Linear Programming to Life Cycle Assessment. PhD Dissertation, University of Surrey, Guildford, UK
- AZAPAGIC, A.; CLIFT, R. (1994): Allocation of Environmental Burdens by Whole-System Modelling – The Use of Linear Programming. In: Allocation in LCA (Huppes, G. and F. Schneider, eds.). SETAC, Brussels, pp 54-60
- AZAPAGIC, A.; CLIFT, R. (1995): Life Cycle Assessment and Linear Programming – Environmental Optimisation of Product System. *Comp. & Chem. Eng.* Vol. 19, Suppl., pp 229-234
- AZAPAGIC, A.; CLIFT, R. (1998): Linear Programming as a Tool in Life Cycle Assessment. *Int. J. LCA* 3 (6) 305-316
- AZAPAGIC, A.; CLIFT, R. (1999a): Allocation of Environmental Burdens in Multiple-function Systems. *J. Cleaner Prod.* 7(2) 101-119
- AZAPAGIC, A.; CLIFT, R. (1999b): Life Cycle Assessment as a Tool for Improving Process Performance: A Case Study on Boron Products. *Int. J. LCA* 4 (3) 133-142
- AZAPAGIC, A.; CLIFT, R. (1999c): Life Cycle Assessment and Multiobjective Optimisation. *J. Cleaner Prod.* 7(2) 135-143
- AZAPAGIC, A.; CLIFT, R. (2000): Allocation of Environmental Burdens in Co-product Systems: Process and Product-related Burdens (Part 2). *Int. J. LCA*, in press
- BOUSTEAD, I. (1992): Eco-balance Methodology for Commodity Thermoplastics. PWMI, Brussels
- CLIFT, R.; AZAPAGIC, A. (1995): System Analysis and Allocation in Multiple-Input Waste Treatment Processes. In: Life Cycle Assessment and Treatment of Solid Waste (Finnveden G. and G. Huppes, eds.), AFR-Report 98, Stockholm, pp 19-25
- CLIFT, R.; AZAPAGIC, A. (1999): The Application of Life Cycle Assessment to Process Selection, Design and Operation. *Proc. of the Advanced NATO Research Workshop on Tools and Methods for Pollution Prevention*. Prague, 12-14 October 1998
- CLIFT, R.; FRISCHKNECHT, R.; HUPPES, G.; TILLMAN, A.-M.; WEIDEMA, B. (eds.) (1998): Toward a Coherent Approach to Life Cycle Inventory Analysis. Report of the Working Group on Inventory Enhancement. SETAC-Europe
- CLIFT, R.; FRISCHKNECHT, R.; HUPPES, G.; TILLMAN, A.-M.; WEIDEMA, B. (eds.) (1999): Summary Report of Working Group on Inventory Enhancement, SETAC Newsletter, Vol. 10, Issue 3, pp 14-20, May 1999, Brussels
- CONSOLI, F. et al. (eds.), (1993): Guidelines for Life-Cycle Assessment: A "Code of Practice". SETAC, Brussels
- DANTZIG, G.B. (1963): Linear Programming and Extensions. Princeton University Press, Princeton
- Dash Associates (1993): XPRESS-MP software. Dash Associates, Blisworth, UK
- DOIG, A.; CLIFT, R. (1995): Applying Life Cycle Thinking to Solid Waste Management. National Conference on Life Cycle Inventory Analysis for Waste Management, ETSU and DOE, Oxfordshire, May 1995
- HEIJUNGS, R. et al. (eds.), (1992): Environmental Life Cycle Assessment of Products: Background and Guide. MultiCopy, Leiden
- FLOUDAS, C.A. (1995): Nonlinear and Mixed-integer Optimization: Fundamentals and Applications. University Press, Oxford
- ISO 14041 (1998): Environmental Management – Life Cycle Assessment: Goal and Scope Definition and Inventory Analysis
- LINDFORS, L.-G.; CHRISTIANSEN, K.; HOFFMAN, L.; VIRTANEN, Y.; JUNTILLA, V.; LESKINEN, A.; HANSEN, O.-J.; RONNING, A.; EKVALL, T.; FINNVEDEN, G. (1995): Nordic Guidelines on Life Cycle Assessment. Nordic Council of Ministers, NORD 1995:20, Copenhagen
- TILLMAN, A.-M.; EKVALL, T.; BAANN, H.; RYDBERG, H. (1994): Choice of System Boundaries in Life Cycle Assessment. *J. Cleaner Prod.* 2 (1) 21-29

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## 6 Appendix: Linear programming and allocation

The main characteristic of Linear Programming (LP) is that it models physical and technical relationships between the inputs and outputs and environmental interventions of the system. Therefore, LP lends itself naturally to allocation according to step (ii) of the ISO procedure (ISO, 1998). Moreover, because LP modelling describes complex interactions among different parts of the system, it can describe changes in the operating state of the system and associated environmental interventions, resulting from changes in product or process properties. This approach therefore reveals how environmental burdens and impacts – and their allocation among different functions – change as the operation of the system is changed.

LP is applicable in cases where physical causality can be described by a linear model of the most general form or can be approximated as linear. Because marginal changes are infinitesimal, a linear model can always be used, for example by linearising a system model about a particular operating state. Incremental and average changes will not normally be linear but under some circumstances can be approximated as such. In these cases, the LP representation may also be appropriate. A general mathematical description of LP and the marginal allocation approach is given elsewhere (AZAPAGIC, 1996; AZAPAGIC and CLIFT, 1998, 1999a).

A LP model of a system takes the general form:

$$\text{Maximise (or minimise)} \quad F = \sum_{i=1}^I f_i x_i \quad (1)$$

$$\text{subject to} \quad \sum_{i=1}^I a_{c,i} x_i = e_c \quad c = 1, 2, \dots, c \quad (2)$$

$$\sum_{i=1}^I a_{c,i} x_i \leq e_c \quad c = c+1, \dots, C \quad (3)$$

and

$$x_i \geq 0 \quad i = 1, 2, \dots, I \quad (4)$$

where eqn. (1) represents an objective function, usually a measure of economic performance, and eqns. (2)-(4) are the constraints in the system, describing material and energy balance relationships, productive capacities, raw material availabilities, market demand and so on. The constraints are, therefore, related to the product and process properties of the system and the parameters,  $e_c$ , represent their limiting values. The (continuous) variables,  $x_i$ , often referred to as activities, represent quantitative measures of material and energy flows including inputs, flows within the economic system, and outputs. They also describe the functional outputs. In the context of LCA, variables  $x_i$  encompass all activities from extraction of the primary materials from the earth through processing to final disposal.

The general LP model given by eqns. (1)-(4) takes the following form for the boron system:

$$\text{Mass balance constraints:} \quad \sum_{i=1}^I a_{c,i} x_i = 0, \quad c = 1, 2, \dots, C \quad (5)$$

$$\text{Market demand constraints:} \quad P_l \leq D_l, \quad l = 1, 2, \dots, L \quad (6)$$

$$\text{Primary material availability:} \quad R_m \leq S_m, \quad m = 1, 2, \dots, M \quad (7)$$

$$\text{Productive capacity constraints:} \quad \sum_{i=1}^I x_i \leq C_u, \quad u = 1, 2, \dots, U \quad (8)$$

$$\text{Heat requirement constraints:} \quad \sum_{i=1}^I H_i \leq Q_z, \quad z = 1, 2, \dots, Z \quad (9)$$

Constraints (6) set limits on the demand for each product; for example, if the functional unit is defined as the operation of the system for one year, the product demand  $D_l$  represents the output of each of the five products in one year ( $\rightarrow$  Table 1). Similarly, constraints (7)-(9) ensure that the consumption of primary and raw materials  $R_m$  does not exceed their supply  $S_m$ ; that flows  $x_i$  are limited by the plant or unit operations capacity  $C_u$ ; and that heat demand  $H_i$  is constrained by the heat availability  $Q_z$ .

In the context of LCA, an economic function  $F$  given by eqn. (1) is replaced by a number of objective functions defined as environmental burdens or impacts (AZAPAGIC and CLIFT, 1995; 1998, 1999c):

$$\text{Minimise} \quad B_j = \sum_{i=1}^I bc_{j,i} x_i, \quad j = 1, 2, \dots, J \quad (10)$$

$$\text{Minimise} \quad E_k = \sum_{j=1}^J ec_{k,j} B_j, \quad k = 1, 2, \dots, K \quad (11)$$

where the coefficients  $bc_{j,i}$  represent resource depletion and emissions from activity  $x_i$  and  $ec_{k,j}$  is a coefficient that relates burdens to the impacts, as defined by the "problem oriented" approach (HEIJUNGS et al., 1992).

In conventional LP modelling, the system is optimised on an objective function to find the optimum operating conditions of the system, which maximises (or minimises) an economic objective function subject to the constraints in the system. In the context of LCA, optimisation on the environmental objectives (10) or (11) identifies the optimum options for improving the performance of the system. The value of LP in Improvement Assessment is discussed elsewhere (AZAPAGIC and CLIFT, 1995; 1998, 1999c). In present work, the objective is to show how system modelling can be used for solving allocation based on physical causality. For these purposes, the system is not optimised – it is only solved for the functions (10) or (11), depending on whether the analysis is carried out at the Inventory or Impact Assessment levels. At the solution of the LP model, total burdens or impacts are calculated together with the marginal or dual values associated with each constraint. Dual values represent the marginal allocation coefficients which relate changes in the burden or impact to a marginal change in one parameter  $e_c$ , while other parameters are held constant (AZAPAGIC and CLIFT, 1999a). At the solution, the burdens and impacts are equal to:

$$B_j = \sum_{c=1}^C \lambda_{j,c} e_c \quad (12)$$

or

$$E_k = \sum_{c=1}^C \mu_{k,c} e_c \quad (13)$$

where  $\lambda_{j,c}$  and  $\mu_{k,c}$  are the marginal or dual values of the  $c$ th constraint, corresponding to the burden or impact objective functions, respectively. The marginal values are defined as:

$$\lambda_{j,c} = \left( \frac{\partial B_j}{\partial e_c} \right)_{e_1, e_2, \dots, e_{c-1}, e_{c+1}, \dots, e_C} \quad (14)$$

or

$$\mu_{k,c} = \left( \frac{\partial E_k}{\partial e_c} \right)_{e_1, \dots, e_{c-1}, e_{c+1}, \dots, e_C} \quad (15)$$

In a co-product system, the parameters  $e_c$  and therefore the allocated burdens  $\lambda_{j,c}$  and impacts  $\mu_{k,c}$ , can be either product- or process-related, depending on which parameter change causes a change in the burdens or impacts. Since the changes considered depend in general on the goal of the study, and the allocated burdens and impacts are dependent on the type of changes analysed, it follows that the allocated burdens will also depend on the goal of the study. If the goal is to analyse changes in the burdens due to a change in a product output, then the burdens and impacts are allocated among the products and are considered to be product-related. In LP terms this means that constraints (6), describing product outputs, are active at the solution of the model (see e.g. DANTZIG, 1963). By definition, their marginal values (which represent the product-related allocated burdens) are non-zero. Operation of the system is now constrained solely by product demand; the availability of raw and primary materials is unlimited and the capacities and heat demand of the operating units do not constrain the production. Therefore, constraints related to the process, i.e. eqns. (7)-(9), are non-active and the process-related burdens and impacts are zero.

For the product-related burdens and impacts, each system parameter  $e_c$  is defined by the output of product  $D_l$  (eqn. (6)), so that the allocated burdens are equal to:

$$\lambda_{j,c} = b_{j,l} = \left( \frac{\partial B_j}{\partial D_l} \right)_{D_1, \dots, D_{l-1}, D_{l+1}, \dots, D_L} \quad (16)$$

and the total burdens are then given by:

$$B_j = \sum_{l=1}^L b_{j,l} P_l \quad (17)$$

Similarly, the allocated and the total impacts are defined by:

$$\mu_{k,c} = ei_{k,l} = \left( \frac{\partial E_k}{\partial D_l} \right)_{D_1, \dots, D_{l-1}, D_{l+1}, \dots, D_L} \quad (18)$$

$$E_k = \sum_{l=1}^L ei_{k,l} P_l \quad (19)$$

Thus, the burdens and impacts related to the activities  $x_i$ , as defined by eqns. (10) and (11), have been translated into the burdens and impacts related to the product outputs  $P_l$ . It should be noted that, under these circumstances, the burdens and impacts are fully allocated among the products, in agreement with the recommendation of ISO 14041 (ISO, 1998). It is then possible to evaluate the contribution of each co-product to the total environmental burden and impacts. Product-related burdens for the boron system are discussed in Section 3.2.

However, if the goal of the study is to consider the effects of changes in the process-related parameters, then the burdens are allocated among these parameters and treated as process-related (see AZAPAGIC and CLIFT, 2000). In that case, some or all of the process constraints (7)-(9) become active, so that the process-related parameter  $e_c$  is defined by the material availability  $S_m$ , capacity limit  $C_u$  or heat availability  $Q_z$ . The allocated burdens are then equal to:

$$\lambda_{j,c} = b_{j,m} = \left( \frac{\partial B_j}{\partial S_m} \right)_{S_1, \dots, S_{m-1}, S_{m+1}, \dots, S_M} \quad (20)$$

or

$$\lambda_{j,c} = b_{j,u} = \left( \frac{\partial B_j}{\partial C_u} \right)_{C_1, \dots, C_{u-1}, C_{u+1}, \dots, C_U} \quad (21)$$

or

$$\lambda_{j,c} = b_{j,z} = \left( \frac{\partial B_j}{\partial Q_z} \right)_{Q_1, \dots, Q_{z-1}, Q_{z+1}, \dots, Q_Z} \quad (22)$$

where some or all marginal burdens can be non-zero, depending on which constraints are active. The total burdens are then equal to:

$$B_j = \sum_{m=1}^M b_{j,m} S_m + \sum_{u=1}^U b_{j,u} C_u + \sum_{z=1}^Z b_{j,z} Q_z \quad (23)$$

Similarly, the allocated impacts are equal to:

$$\mu_{k,c} = ei_{k,m} = \left( \frac{\partial E_k}{\partial S_m} \right)_{S_1, \dots, S_{m-1}, S_{m+1}, \dots, S_M} \quad (24)$$

or

$$\mu_{k,c} = ei_{k,u} = \left( \frac{\partial E_k}{\partial C_u} \right)_{C_1, \dots, C_{u-1}, C_{u+1}, \dots, C_U} \quad (25)$$

or

$$\mu_{k,c} = ei_{k,z} = \left( \frac{\partial E_k}{\partial Q_z} \right)_{Q_1, \dots, Q_{z-1}, Q_{z+1}, \dots, Q_Z} \quad (26)$$

and the total impacts are defined by:

$$E_k = \sum_{m=1}^M ei_{k,m} S_m + \sum_{u=1}^U ei_{k,u} C_u + \sum_{z=1}^Z ei_{k,z} Q_z \quad (27)$$

Finally, in a study where changes in both parameters are of interest, the allocated burdens are considered to be both product- and process-related, and the total burdens and impacts are given by:

$$B_j = \sum_{l=1}^L b_{j,l} P_l + \sum_{m=1}^M b_{j,m} S_m + \sum_{u=1}^U b_{j,u} C_u + \sum_{z=1}^Z b_{j,z} Q_z \quad (28)$$

and

$$E_k = \sum_{l=1}^L ei_{k,l} P_l + \sum_{m=1}^M ei_{k,m} S_m + \sum_{u=1}^U ei_{k,u} C_u + \sum_{z=1}^Z ei_{k,z} Q_z \quad (29)$$

Marginal allocation and changes from product- to process-related burdens in the boron system are discussed in Azapagic and Clift (2000).

## Nomenclature

$a_{i,j}$	Input/output coefficients of a process or activity $i$	$e_c$	Right hand side coefficients or parameters of the constraints
$bc_{k,i}$	Environmental burden coefficients	$E_k$	Total Environmental impact
$b_{i,j}$	Marginal burden allocated to the co-products	$f_i$	Coefficients in the economic objective function
$b_{i,m}$	Marginal burden allocated to the material availability	$F$	Economic objective function
$b_{i,u}$	Marginal burden allocated to the capacity	$H_i$	Heat requirement in the system
$b_{i,v}$	Marginal burden allocated to the heat requirement	$P_i$	Product output
$B_i$	Total Environmental burden	$Q_c$	Heat availability
$C_u$	Capacity of a process of an operation unit	$R_m$	Primary or raw material availability
$D_i$	Market demand on the output of the products	$S_m$	Supply of primary or raw material
$ec_{k,i}$	Environmental impact coefficients	$x_i$	Output from a process or activity (operation level)
$ei_{k,m}$	Marginal impact allocated to the material availability	$\lambda_{ic}$	Marginal or dual value of the $c$ th constraint; equal to the marginal allocated burden
$ei_{k,u}$	Marginal impact allocated to the capacity	$\mu_{kc}$	Marginal or dual value of the $c$ th constraint; equal to the marginal allocated impact
$ei_{k,i}$	Marginal impact allocated to products		
$ei_{k,v}$	Marginal impact allocated to the heat requirement		

## Forthcoming (No. 1, 2000): Part 2

## Allocation of Environmental Burdens in Co-product Systems: Process and Product-related Burdens (Part 2)

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**Abstract.** ISO 14041 requires that allocation by physical causality must reflect the quantitative changes in product outputs or functions and will not necessarily be in proportion to simple physical measure such as mass. This paper examines the instances where physical causality can be represented by mass. However, it also goes further than ISO to demonstrate that the type of causality in the system is not necessarily always the same and can change depending on the way the system is operated. Whole system modelling and the marginal allocation approach are used to identify the correct type of causality for different operating states of the system and the corresponding changes in the environmental burdens. This is generally not possible with the other allocation methods, also examined in this paper. Both process- and product-related burdens are considered and the approach is illustrated by a reference to an existing system producing five boron co-products.

**Keywords:** Allocation; boron; environmental impacts; LCA; Life Cycle Assessment; linear programming; marginal values; system analysis

## Introduction

ISO 14041 (1998) set guidelines for dealing with allocation in multiple-function systems. According to this proce-

duce, allocation should be avoided where possible by system disaggregation or by expanding system boundaries. If that is not feasible, then the environmental burdens should be partitioned among different functions of the system in a way which reflects the underlying physical causality. This implies that the allocated burdens must follow the quantitative changes in product outputs or functions, which will not necessarily be in proportion to simple physical measure such as mass.

This paper focuses on allocation by physical causality, the second step in the ISO 14041 hierarchy. However, it goes further than ISO to demonstrate that the type of physical causality and, hence, the allocated burdens in a system are not necessarily fixed but can change depending on the way the system is operated. Marginal allocation and whole system modelling (AZAPAGIC and CLIFT, 1998, 1999a) are used to identify the "active" or relevant physical causality and allocation parameters in the system. These considerations are illustrated by a "cradle to gate" study of an industrial multiple-function system producing boron co-products (AZAPAGIC and CLIFT, 1999b).